

Financial Science Trends and Perspectives: A Review Article

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Abstract

Robert C. Merton's contributions are current. Regarding the design of retirement plans, in a framework of deficient pension systems, the investigations of Merton (1969) and (1971) on optimal consumption and portfolio rules during and after working life acquire contemporary validity. Likewise, Bodie and Merton (2002) propose the use of derivative products, at the international level, to diversify the risks of pension systems just at the moment when these systems of many underdeveloped and industrialized economies are on the verge of collapse; knowing that several of these systems only provide a meager proportion of the salary. Finally, Merton's theory of rational option pricing is retaken to create synthetic oil pipelines and power plants through the use of contingent claims. This paper aims to review the trends and perspectives in financial science and mathematical finance within the framework of Robert Cox Merton's pioneering contributions, highlighting priority areas that offer research opportunities with social and global impacts.

JEL classification: D11, G12, G13, E43.

Keywords: financial science, consumer and portfolio decisions, derivative products, interest rate term structures.

Tendencias y perspectivas de la ciencia financiera: Un artículo de revisión

Resumen

Las contribuciones de Robert C. Merton cobran vigencia actual. Con respecto del diseño de los planes para el retiro, en un marco de sistemas deficientes de pensiones, las investigaciones de Merton (1969) y (1971) sobre reglas óptimas de consumo y portafolio durante y después de la vida laboral cobran vigencia contemporánea. Asimismo, Bodie y Merton (2002) proponen el uso de productos derivados, en el ámbito internacional, para diversificar los riesgos de los sistemas de pensiones justo en el momento en que muchos de éstos, en economías subdesarrolladas e industrializadas, están en la antesala de colapso; además de que varios de estos sistemas sólo garantizan una proporción exigua del salario. Por último, la teoría racional de valuación de opciones de Merton (1973) se retoma para crear oleoductos y plantas generadoras de energía sintéticas mediante el uso de contratos derivados. Este trabajo pretende realizar una revisión de las tendencias y perspectivas de la ciencia financiera y las finanzas matemáticas, en el marco de las contribuciones pioneras de Robert Cox Merton, resaltando áreas prioritarias que ofrecen oportunidades para realizar investigación con impactos social y global.

Clasificación JEL: D11, G12, G13, E43.

Palabras clave: ciencia financiera, decisiones de consumo y portafolio, productos derivados, estructuras de plazos de tasas de interés.

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1. Introduction

A relevant aspect of Robert Cox Merton's contributions is that they are all current today. For example, on the subject of retirement plans design, Merton's (1969) and (1971) research on optimal consumption and portfolio rules during and after working life are currently valid, especially in the knowledge that the today pension systems only provide a meager proportion of the salary (Merton, 2006). The use of derivative contracts is also current at the international level to diversify the risks in pension systems without modifying the patterns and behaviors of the employment, the industrial structure and the functioning of the financial system (Bodie and Merton, 2002). In addition, the work of Merton (1973) takes up in an updated way his rational theory of pricing options to create synthetic pipelines and power plants through the use of derivative products to make the use of energy resources more efficient and, with it, have a greener world; it should be noted that the link between energy and the environment is one of the most important worldwide.

This article aims to examine the trends in financial sciences and mathematical finance based on the contributions of R. C. Merton that, surprisingly, are still current. A review of some other recent contributions on the management of various risks is also made. Finally, the areas that offer research opportunities with social and global impact in the framework of the pioneering contributions of Robert Cox Merton are highlighted.

This work is organized as follows: section 2 briefly describes the evolution and current relevance of R. C. Merton's contributions; section 3 discusses the stimulus financial innovations gave financial science to mature, highlighting the contribution of derivatives and emphasizing how they can be used now to make energy policymaking more efficient, and also stressing how derivatives can be used to diversify risks in pension systems; section 4 presents other contributions on the management of various risks; section 5 establishes lines of future research with social and global impact based on Merton's contributions; finally, section 6 gives the conclusions

2. The Evolution of R. C. Merton's Contributions

Merton (1973), in his pioneering article, deduces a set of necessary conditions that the premium of an option must satisfy. The requirements are based on the assumption that investors prefer more to less (say, a rational theory of prices). Merton also focuses on underlying assets that pay dividends and when the terms of the option contract may be changed explicitly by an adjustment in the exercise price or implicitly by changes in the company's investment policy or capital structure. Since the conditions proposed by Merton are not sufficient to uniquely determine the premium of an option, Merton makes additional assumptions to extend the Black-Scholes formula (1973). Merton obtains explicit formulas for the European call premium, warrants and "down-and-out" options in his work. Merton (1973) also examines the effects of dividends on the price of the collateral. He also extends the theory of pricing of corporate liabilities. Other no less essential contributions of Merton in his 1973 article are the analysis of American options, the generalization of the Samuelson formula for perpetual options, the valuation of options with barriers, and the

introduction of stochastic interest rates. An essential difference with Black and Scholes's (1973) classical model is that Merton does not assume a constant interest rate free of default risk, but the returns of a zero-coupon bond driven by a diffusion process, which generates a term structure for the interest rate.³

Two other pioneering articles by Robert C. Merton (1969) and (1971) deal with a rational consumer that maximizes his/her total discounted utility when a diffusion processes drives the asset returns dynamics.⁴ Undoubtedly, a current issue is retirement plans in which agents must follow optimal consumption and portfolio rules during working life, foreseeing that pension systems only guarantee a meager proportion of salary (Merton, 2006). In this sense, Merton makes the following recommendations to plan retirement: 1) consider the risks associated with the returns of what is expected to be received; 2) determine an investment strategy that assumes the risk you want to expose yourself to; 3) determine the optimal rules on assets before and after you retire; 4) take into account market changes; 5) propose a low-cost investment strategy (commissions).⁵

On the other hand, a large part of the extensions of the works of Merton (1969) and (1971) point towards the modeling of the stochastic behavior of asset prices. For example, when they occur with regime-switching trend or volatility defined by a Markov chain. In this sense, Bäuerle and Rieder (2004) determine the optimal portfolio's optimal decisions when the share price depends on a finite Markov chain and homogeneous in time. Sotomayor and Cadenillas (2009) find explicit solutions for optimal investment and consumption decisions with a HARA (Hyperbolic Absolute Risk Aversion) utility function when asset prices are driven by standard Brownian movements combined with a Markov-switching model. Finally, Fei (2013) provides the optimal consumption and portfolio decisions when a Markovian regime change process drives the inflation rate.

Other works, not less important that have studied the problem of determining optimal consumption and portfolio decisions with Markov-switching models are, for example, Stockbridge (2002) that provides a mathematical programming formulation of the portfolio optimization problem; Zhang and Yin (2004) who offer near-optimal strategies in a financial market; Sass and Haussmann (2004) who solve the problem of maximizing the expected utility of the investor of the final wealth in a finite time horizon numerically; and Vallejo-Jiménez et al. (2015) and Soriano-Morales et al. (2015) that combine Markov-switching models with multiple jumps.⁶

³ It is important to note here that Black and Scholes (1973), under a set of assumptions, determine the premium of a European call option following different ideas from Merton (1973). Black and Scholes (1973) use the heat diffusion equation and the Capital Asset Pricing Model (CAPM) Theory.

⁴ Optimal portfolio selection has been extensively studied in Biagini and Øksendal (2003), Czichowsky and Schachermayer (2015), Hu and Øksendal. (2003), Hu et al. (2003), Jumarie (2005), Karatzas et al. (1987), He and Pearson (1991), Karatzas et al. (1991), Cvitaní' and Karatzas (1996), Cvitaní' and Wang (2001), Venegas-Martínez (2001), (2005) and (2009), Venegas-Martínez and González-Aréchiga (2000), and Zariphopoulou (2001), (1999) and (1992).

⁵ In the link https://scholar.google.es/scholar?start=10&q=robert+c+merton&hl=es&as_sdt=0,5 appear all the publications of Robert C. Merton.

⁶ As far as we know, regime change models were initially proposed by Hamilton (1990). Furthermore, the first to deal with asset prices driven by mixed diffusion processes with jumps were: Cox and Ross (1976), Ball and Torous (1985), and Page and Sanders (1986). More recent work on the jump-diffusion process can be found in Ait-Sahalia et al. (2009) and Lui et al. (2005). The fractional Brownian motion is a natural extension of Brownian motion (Mandelbrot, 1968), and its statistical properties are widely used in financial modeling; see, for example, Bender et al. (2011) and Hu and Øksendal (2003).

It is worth making a small list that highlights a part of the theoretical development that Merton's pioneering work (1969) and (1971) has promoted in the specialized literature on the rational consumer-investor in continuous time with asset returns driven by broadcasts. Of course, this list, in Table 1, is by no means intended to be exhaustive.

Table 1. The impulse of the pioneering work of Merton (1969) and (1971).

Diffusion process.	Merton (1969) and (1971)
Jump-diffusion process.	Czichowsky and Schachermayer (2015), Jin and Zhang (2012), Aït-Sahalia <i>et al.</i> (2009), Venegas-Martínez (2000) and (2001), Jeanblanc-Picqué y Pontier (1990).
Diffusion process modulated by a finite Markov chain homogeneous in time.	Soriano-Morales <i>et al.</i> (2015), Fei (2014), Zhou y Yin (2014), Wu y Li (2011), Elliott <i>et al.</i> (2010), Sotomayor and Cadenillas (2009), Çakmak and Özekici (2006), Rieder and Bäuerle (2005), and Bäuerle and Rieder (2004), Sass (2004), Sass and Hausmann (2004), Stockbridge (2002), and Elliot (2002).
Diffusion process modulated by a finite Markov chain not homogeneous in time.	Vallejo-Jiménez <i>et al.</i> (2015), and Rudiger and Backhau (2008).
Fractional Brownian motion modulated by a homogeneous finite Markov chain in time.	Fei and Shu-Juan (2012).
Jump diffusion process modulated by a Markov chain.	Yu (2014), and Elghanjaoui and Karlsen (2012).
Fractional Brownian motion combined with multiple processes - jumps modulated by a not homogeneous Markov chain in time.	Vallejo-Jiménez and Venegas-Martínez (2017).

Source: Authors' elaboration.

3. Financial systems, innovations, and financial science

For the best functioning of financial systems, financial innovation is essential, which increases the efficiency of these systems through new financial instruments and the use of novel models, techniques, tools, and methodologies, all together with the advancement of information technologies.

New proposals for various models in the 1950s and 1960s aided finance to become a formal science. Subsequently, the creation of multiple instruments inexorably connected financial science

and financial practice in the 1970s and 1980s, among which derivative products stand out.⁷ Some groundbreaking events that helped make finance a science between 1950 and 1960 include the following:

- 1952, diversification through portfolio theory with Harry Markowitz's mean-variance approach.
- 1953, The Role of Financial Assets in Optimal Risk Allocation by Kenneth Joseph Arrow.
- 1958, James Tobin's Liquidity Preference as Risk Behavior.
- 1958, The Optimal Capital Structure in Corporate Finance and Dividend Policy by Franco Modigliani and Merton H. Miller.
- 1960, the first comprehensive database of individual stock performance from the Chicago Asset Price Research Center.
- 1963 and 1965, the Efficient Market Hypothesis of Eugene Fama and Paul Samuelson.
- 1964, 1965, and 1966, William Forsyth Sharpe - John Lintner - Jan Mossin - Jack L. Treynor risk-based differences in expected returns, and the Capital Asset Pricing Model (CAPM).
- 1968 and 1969, various institutional investors' performance tests using the CAPM - Michael C. Jensen and Richard Roll.

On the other hand, crises can induce the generation of financial innovations that lead to permanent improvements in the financial system, as happened in the 1970s and 1980s. One of the major financial and economic crises in history occurred in the 1970s in the United States with the following characteristics:

- The multidimensional explosion of volatility in Western economies is reflected in financial systems.
- The fall of the Bretton Woods monetary system.
- The first oil crisis in 1973-1974 and a second in 1979.
- Double-digit inflation in US, the highest since the Civil War.
- Double-digit interest rates, the highest since the Civil War.
- Lack of availability of mortgage funds: the Q-5% regulation and the interest payment limit on deposit accounts.
- "Stagflation" as an economic disease, unknown and still without a solution.
- Fall of the Stock market by 50% in real terms in 1973-1974.
- The 1973-1975 recession and its adverse effects until the 1980s.

This explosion of risk in the US in the 1970s led to an explosion of financial innovations, later adopted worldwide. During this period, financial practice and theory are linked, as shown by the following facts:

- Over-the-counter market for options.
- Financial futures for currencies, interest rates, and stocks.

⁷ It is worth mentioning here another significant contribution on stochastic economic growth provided by Merton (1975).

- NASDAQ, the first automated and electronic stock market.
- Money Market Funds, High Yield Floating Rate Bonds.
- Index funds, Stage Coach Fund 1970 and Vanguard 1975.
- International diversification of shares of TIAA-CREF 1972.
- Modern pension system ERISA 1974, financed by employers in the U.S.A.
- On May 1, 1975, the collection of variable commissions on the purchase and sale of shares was allowed.
- Debt securitization and creation of a national mortgage market.
- Elimination of destructive regulations: maximum limits on deposit rates.
- Laying the foundations for the globalization of capital markets: derivatives markets adopted almost worldwide and, through them, global diversification.
- Financial science maturity: Both existing and novel quantitative models were used widely in financial systems.
- The first departments (or faculties) of finance and undergraduate and postgraduate degrees in finance appear.

An important question regarding these innovations is how intangible derivative contracts can solve real economic growth challenges and stabilization? Derivatives contracts can redistribute risks in a non-invasive and reversible way. Here are some special features of the relationship between derivatives markets and financial innovation, and of course, we highlight the maturation of financial science that has occurred with derivatives.

- Derivatives are efficient “adapters” between heterogeneous financial systems that improve global financial integration and diversification.
- Derivatives provide an efficient implementation of the three risk management methods: diversification, hedging, and insurance.
- The development of derivatives markets for stocks, interest rates, currencies, raw materials, and primary goods (commodities) promotes financial stability through multiple risk transfer and price information extraction channels.
- Derivatives can improve stabilization and open market operations efficiency: efficient trading and issuance of “open market policy” securities.
- Informed regulation to reap the benefits of financial innovation while managing risks.

3.1 Some success stories of increased efficiency and a cleaner world

For decades, contracts have been replaced by physical assets to create greater efficiency and a cleaner world, as were the cases of the Leipzig Gas Pipeline in Germany, and the Tennessee Valley, in US, the largest public energy company in the US, such as and as shown below. These experiences have to be taken up again today to make energy resources more efficient and have a greener world.

3.1.1 Leipzig gas pipeline

Consider the Leipzig gas pipeline in the 1990s, where derivative contracts that played the role of a synthetic pipeline were used to create a more efficient energy source in a cleaner world. German reunification in 1990 generated rapid economic development and increased demand for energy. To satisfy this demand, a greater supply of natural gas was required. Leipzig had two options shown in Figure 1 and Table 1.

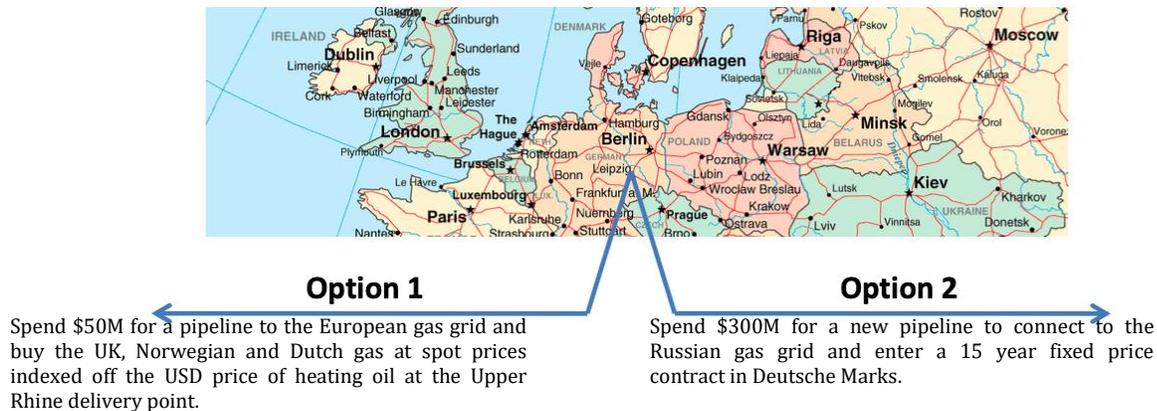


Figure 1. Leipzig gas pipeline options.

Table 2. Contractual asset synthesis: Leipzig gas pipeline.

	Option 1	Option 2
Capital Investment	\$50M	\$300M
Advantages	Reduced political risk by avoiding dependence on Russians. Lower capital investment.	Stable prices of power potentially useful to a population accustomed to price controls.
Disadvantages	Gas price volatility.	High capital investment.

Option 1 could be made attractive with hedging but had two significant problems:

- Limited hedge instruments available:
 - 2. Crude oil call up to 5 years in USD.
 - 3. Crude/heating oil basis swaps up to 2 years.
 - 4. F.X. Options up to 5 years.
 - 5. Currency swaps up to 10 years.
- Limited sophistication of the city administration.



Efficient and Green Solution

A bank provided a 15-year cap on European gas prices at a strike price equal to the Russian fixed-price contract in exchange for a premium of \$125 MM. The cap is effectively a “synthetic pipeline.”

The price is half of the incremental cost of a physical pipeline to Russia and compensates the bank for hedge mismatches and the need to adjust hedges over 15 years dynamically.

3.1.2 The Tennessee Valley Authority

Another case that should be highlighted is that of the Tennessee Valley Authority (T.V.A.), which used purchase option contracts to buy energy as an alternative to the construction of new generating plants. In other words, the options contracts played the role of generating plants. In this way, T.V.A. did not build two nuclear power plants, equivalent to 35 conventional ones.

- In 1994, the Tennessee Valley Authority (T.V.A.), the largest public energy company in the US, carried out a long-term strategic analysis of its customers' energy demands and developed robust supply channels that were not affected by external shocks for the flexibility offered.
- T.V.A. adopted an innovative proposal to meet incremental capacity needs by using derivative contracts to purchase energy as an alternative to the construction of new generating plants.
- T.V.A.'s proposal consisted of creating long-term purchase option contracts on energy with counterparties that could deliver power to the grid.
- As a consequence of the agreement's implementation, T.V.A. did not build two nuclear power plants, equivalent to 35 conventional ones.
- In 2016, T.V.A. was responsible for 3.5% of all-electric power generation in the U.S.

3.2 International Pension Swaps

Lessons from international experience have shown various benefits of swaps to diversify the risk of pension systems. These benefits can be listed as follows:

- Lower cost of capital through increased global risk acquisition of a country's risks.
- Natural counterparts always available: If a country has "too much" risk exposure for itself through diversification, the rest of the world has "too little" risk exposure in that country.
- Low-cost implementation. Transactions are carried out directly between sovereign wealth funds, government pension funds, reserves, and central banks, without the need to incur intermediation costs or credit risk.
- Reduction of the moral hazard of expropriation, repudiation, taxes, or accounting.
- They grant a credit guarantee and/or two-way collateral with market value.

In recent years, derivative products, particularly swap contracts, have become financial devices that connect various national financial systems with the global financial network. Today, financial systems worldwide widely use swaps (interest rates and exchange rates) to manage their market risks and lower their transaction costs (commissions). However, the vast majority of pension funds that have increased over the past 20 years rarely use swaps when they could use them to diversify their risks internationally. The relative advantages offered by swaps in the coverage of pension funds are:

- Residents conduct industrial governance, local market share trading, and regional brokerage distribution of exposures to global asset returns.
- The use of these derivatives is robust concerning the local financial system's design: it works with financial stabilization policies (including capital controls).
- The policy is non-invasive: it does not require changes in employment patterns and behavior, changes in the industrial structure, or changes in the financial system's design.
- The policy is reversible simply by entering an offsetting swaption.

4. Other recent contributions on risk management

Money laundering, not only from criminal activities but also from the diversion of public resources, is one of the present problems. In this regard, the work of Martínez-Sánchez, Cruz-García, and Venegas-Martínez (2020) aims to develop a regression tree helpful model to quantify the risk associated with money laundering (ML) considering the client's profile and its contracted products (inherent risk of the client). Several institutions and entities are exposed to ML risk, but mainly financial ones, due to their activity's nature, so that they are legally obliged to have an adequate methodology to analyze and evaluate said risk. The research carried out uses the regression tree technique (a data science technique) to identify, measure, and quantify the client's inherent ML risk. After classifying customers as high or low risk based on a probability threshold of 0.5. The study finds that: clients with 56 months or more seniority are riskier than those with less seniority; the variables "contracted product" and "customer age" are statistically significant; the legal entity and economic activity are not statistically significant to classify clients; commercial products and individual products are the riskiest; and the percentage of effectiveness, suggested by the decision tree technique, is around 89.5%. In ML risk management's daily practice, the two main problems are 1) the client's knowledge and 2) the detection of its inherent risk elements.

Bernal-Ponce, Castillo-Ramírez, and Venegas-Martínez (2020) investigate the effect of derivatives on the relationship between exchange rates and securities markets. The work uses a rational investor's theoretical model and performs an empirical analysis for the Mexican and Brazilian stock exchanges in 2007-2019. The theoretical model results are applied to international portfolios' management, proposing a strategy to mitigate foreign exchange exposure with derivatives. This study contributes to the literature by explaining how the minimum variance coverage index plays an essential role in the nexus of foreign exchange and stock markets.

Finally, the efficient and transparent granting of microcredits through digital platforms to individuals who carry out economic activities and who seek to maintain their employment and that of their workers and who do not have access to the conventional financial system is undoubtedly a problem. Urgent to be resolved in the global health crisis. The work of Rodríguez-García and Venegas-Martínez (2020) develops various credit risk models and strategies that allow promoting credit inclusion in Mexico fairly and sustainably in an environment of uncertainty generated by the present and expected ravages of the pandemic COVID-19. Thus, machine learning's data science

approach is used, particularly regression trees, random forests, Boosting, K-Nearest Neighbor (K.N.N.), and neural networks.⁸

5. Future research proposals with social and global impacts

Based on the development of the previous sections, some of the areas that offer opportunities for research with social and global effects are presented below in the framework of Merton's pioneering contributions; the list is not intended to be exhaustive,

- Energy derivatives to generate oil pipelines, gas pipelines, and “synthetic” power generating plants with respect for the environment.
- Optimal rules of consumption and portfolio with transaction costs.
- Optimal consumption and portfolio rules with Lévy processes combined with Cox processes modulated by Markov chains.
- Optimal rules for consumption and portfolios during and after an individual's working life.
- Risk diversification in pension systems with swaps in the international arena.
- Risk management with multiple microcredit factors in the environment of the COVID-19 pandemic.
- Development of stochastic macroeconomic models with the health sector and its vulnerability to extreme events (catastrophes, pandemics -COVID-19-, et cetera.)
- Effects of uncertainty in economic policy in financial markets.
- Control of the risk of money laundering with multiple factors.
- Nexus between the financial and energy sectors to have a greener world.
- The behavior of financial agents (behavioral finance) and artificial intelligence.
- Data science applied to banking and business.
- Analysis of the effects of the COVID-19 pandemic on financial markets with data science tools.

6. Conclusions

In the course of this work, a review of the trends and perspectives of financial science and mathematical finance was made based on the contributions of R. C. Merton, which remain current, as well as some other recent contributions in risk management. A detailed follow-up was also made of financial science's evolution and the importance of financial innovation, particularly derivative products, which have increased financial systems' efficiency and the advancement of information technologies. Finally, the research presents a diagnosis of the areas that offer research opportunities with social and global impact.

⁸ In the league.

<https://econpapers.repec.org/scripts/search.pf?ft=&adv=true&wp=on&art=on&bkchp=on&soft=on&pl=&auth=on&sort=rank&lgc=AND&aus=venegas&ar=on&kw=&jel=&nep=&ni=&nit=epdate>
There are about 20 books by F. Venegas-Martínez related to Finance issues.

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