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Proposals to Transfer Risks in Avocado Load

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Abstract

This study aims to mitigate some risks faced by established avocado industry actors in Michoacan by transferring risks from orchards to packaging facilities and distribution points. Using stochastic dynamic games and Merton-Black-Scholes theory, the study aims to maximize the long-term market share of packing house owners in the state in the presence of organized crime. To the best of our knowledge, this is the first time these methods have been used to this end. The premium to be paid in exchange for property rights over turnpikes is defined. Packaging facility owners are recommended to build a branch office in strategic locations to reduce organized crime control. At the same time, the paper suggests highway concessionaires issue insurance against theft of goods and finance it through the proposed procedure. The study includes cost-benefit sketches from both pecuniary and environmental perspectives but acknowledges that a thorough analysis requires knowledge of the local entrepreneurs' willingness to implement green actions.

JEL Classification: C73, G13, G22.
Keywords: Poisson process, Contingent claims, Mean-field game, Oligopoly in a class of systems of interacting objects, Voronoi game

Propuestas para transferir riesgos en el transporte de carga de aguacate

Resumen

Este estudio tiene como objetivo mitigar algunos riesgos que enfrentan los actores establecidos de la industria del aguacate en Michoacán al transferir los riesgos de las huertas a las plantas de empaque y puntos de distribución. Utilizando juegos dinámicos estocásticos y la teoría de Merton-Black-Scholes, buscamos maximizar la participación de mercado a largo plazo de los propietarios de plantas empacadoras en el estado en presencia del crimen organizado. Hasta donde sabemos, esta es la primera vez que se han utilizado estos métodos para este fin. Se define la prima que se debe pagar a cambio de los derechos de propiedad sobre las autopistas. Se recomienda a los propietarios de las plantas de empaque construir sucursales en ubicaciones estratégicas para reducir el control del crimen organizado. Al mismo tiempo, sugerimos que los concesionarios de carreteras emitan seguros contra el robo de mercancías y los financien a través del procedimiento propuesto. El estudio incluye bosquejos de costo-beneficio desde perspectivas pecuniarias y ambientales, pero reconoce que un análisis exhaustivo requiere conocer la disposición real de los empresarios locales para implementar acciones ecológicas.
Clasificación JEL:C73, G13, G22.
Palabras clave: Proceso de Poisson, reclamos contingentes, juego de campo medio, oligopolio en una clase de sistemas de objetos interactuantes, Juego de Voronoi

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1. Introduction

This work intends to provide two (partial) solutions to transfer the risks taken by the established actors in the avocado industry in the Mexican state of Michoacán when they move the product from the orchards to the packaging facilities, or from such facilities to other points of distribution. Indeed, this state is known to have a very serious problem with crime, and both the general population and avocado entrepreneurs must deal with the current wave of violent felonies that disrupt their everyday lives.

To this end, we use mean-field game theory to solve a nonzero-sum game on a system of interacting objects and thus propose a method to maximize the market share of the owners of the packaging facilities under a long-run criterion. We also apply a standard technique from the theory of contingent claims to price a financial instrument for funding highway insurance for the transportation of basic materials. The United Nations Sustainable Development Group (2016) that this project addresses are:

- Good health and well-being.
- Decent Work and Economic Growth.
- Industry, Innovation, and Infrastructure.
- Sustainable Cities and Communities.

2. Goals of the research

General goal. Identify affordable methodologies to:

- Compute the fair price of a toll road insurance investment.
- Encourage the competition among packaging house owners to facilitate access to avocado orchard owners.

Specific goals.

- Isolate algorithms that allow carrying out the valuation of highway insurance.
- Elaborate examples under theoretically accessible hypotheses.

3. Theoretical framework

3.1. Importance of the avocado industry

The Persea Americana is a medium-sized, evergreen tree from the family of Lauraceae. Its fruit is a berry which is suitable for human consumption. As it is studied in the paper by Gutiérrez-Contreras, Lara-Chávez, Guillén-Andrade, and Chávez-Bárcenas (2010), it is originally from the Mesoamerican region, it is largely produced in Brazil, the Dominican Republic, Israel, Colombia, and Mexico; and, according to p.71 in the work by Zentmyer (1998), it constitutes one of the greatest contributions from the Americas to the world. According to the Food and Agriculture Organization of the United Nations (2023), for over a decade now, Mexico has been the main producer of palta in the world. In

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fact, in the last three years, the United States of America has imported around one million tons of *l'or vert du Mexique* per year. As is argued by Maubert (2023), such imports come from the only commercial partner authorized by the United States Department of Agriculture to export aguacateros to the superpower: Mexico. Gutiérrez-Contreras et al. (2010) have proved that the most important Mexican producer and exporter of palto is the state of Michoacán. Figure 1 displays the location of Michoacán in the world, along with the main municipalities that produce the berry.

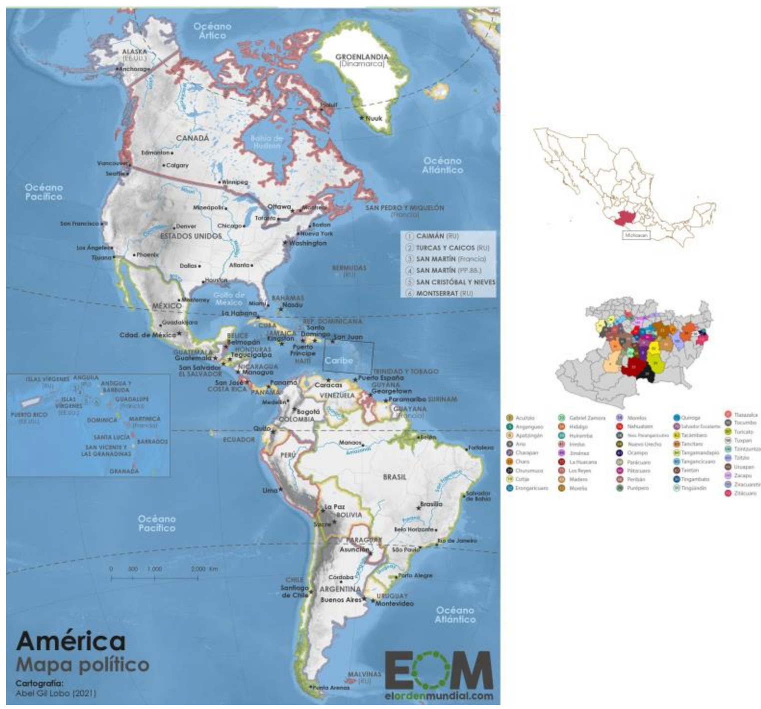

Figure 1. Location of municipalities from Michoacán that produce aguacates. Sources: Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C. (2023); Gil Lobo (2021); Secretaría de Comunicaciones y Transportes (2023).

3.2 Logistics of the avocado industry

According to the Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C. (2023), and its marketing department Avocado from Mexico, the logistics of the production of curo is as in table 1. Currently, six national programs are devoted to incentivizing this fruit's production in Mexico (see section 7.3 in the work by Cámara de Diputados–LXIII Legislatura and Centro de Estudios para el Desarrollo Rural Sustentable y la Soberanía Alimentaria (2017)). All these initiatives support the tasks associated with the first five items from the above list. In addition to these programs, according to Lara (2024), the local authorities have conducted their efforts to the signature of trade treaties that ensure the diversification of the connectivity of the state with other entities or nations.

1. Preparation of soil	6. Transportation to packaging facilities
2. Sowing	7. Emballage
3. Adding manure and fertilizers	8. Transportation to export ports or domestic transportation
4. Irrigation	9. Distribution in retail, wholesale stores, and other sale points.
5. Harvesting	10. Consumption

Table 1. Logistics of the production of aguacates.

A trade treaty is an agreement established by two or more countries under the protection of international law to improve their relations in economic terms and trade exchange. Burges (2013) has extensively documented the international accords that the United States of America (U.S.A.) have implemented to prevent criminal actions against their freighters on their own soil. Currently, Mexico has 14 international trade agreements in force, with the one signed with the U.S.A. and Canada being the most extensive and oldest since it was signed in 1992 and entered into force in 1994. The other countries with which Mexico has signed treaties are Colombia (1995), Chile (1999), Israel (2000), the European Union (2000), Iceland, Liechtenstein, Norway and Switzerland (2001), Uruguay (2004), Japan (2005), Peru (2012), Central America (2013), Panama (2015), and Oceania (2016) (Secretaría de Agricultura y Desarrollo Rural, 2019). Unfortunately, none of these treaties is related to the prevention of cargo theft. The sort of connectivity the local authorities seek in the trade treaties is an airborne way out of Michoacán. However, according to the owner of one of the largest orchards in the state (who asked for their name to be undisclosed for security reasons), a truck (carrying up to nine tons of fruit) needs to drive for up to four hours to reach a packaging facility from the orchards. This represents a problem perse, but the fact that there are no federal credits, fiscal incentives, or international trade agreements that motivate the packaging companies' owners to bring their businesses closer to the producers worsens the situation. This is one factor that enables criminals to hijack the cargo and deprive the established industry of their due revenues.

It is worth mentioning that, once the transporter reaches the packaging facilities from the orchard, they are no longer responsible for the cargo, and the risk of moving it is transferred to the packaging house's manager. Another key fact is that by 2021 there were 84 companies devoted to packaging the avocados produced by nearly 35,000 orchards within latitudes 18.75N and 20.1N, and longitudes 101.783333W and 103.216667W (cf. Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C. (2021) and Gutiérrez-Contreras et al. (2010)). The entrepeneur who we interviewed for this article argues that to this day, none of these packaging facilities has more than one facility to perform their tasks. This is a direct contra- diction to the so-called "best practices" in the industry of basic perishable materials from the orchards to packaging houses, which suggests the usage of different sorts of picking trains depending on a categorization of the orchards into distance groups that go from up to 400 meters to "more than 800 meters"

(Agriculture and Horticulture Development Board, 2022; Michaliszyn-Gabry´s, Krupanek, Kalisz, & Smith, 2022).

3.3 Crime in Michoacán

Since 2018, four out of the ten most dangerous criminal organizations in the world (plus some local independent crime cells) have operated on the soil of Michoacán without any effective action taken by the federal authorities. See figure 2.

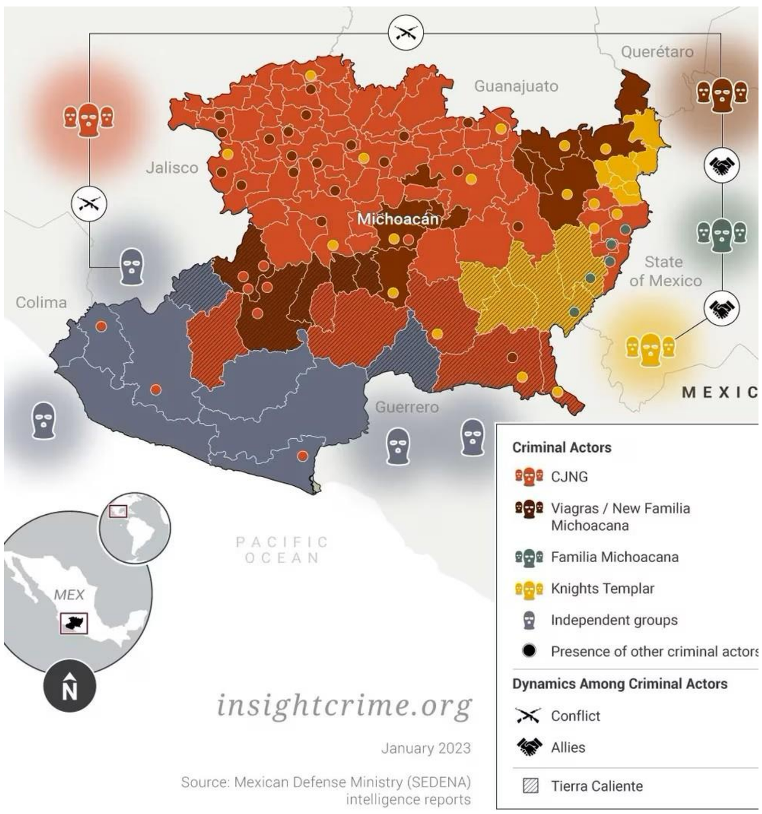

Figure 2. Distribution of criminal cartels on Michoacán. Source: Ashman (2023).

None of these criminal organizations consider the avocado industry as a primary target. They are more interested in other illegal activities (such as drug production and distribution, human and weapon trafficking, and child pornography). However, the avocado industry of Michoacán is forced to endure this harsh environment and must coexist with all these criminal groups, who do not hesitate to steal from them the fruits of their hard work (figuratively and literally speaking).
The crimes referred to in table 2 are the ones that avocado transporters are put against when pirates steal their product when they move their precious cargo. We borrowed the information in such table from the study conducted by the Subsistema Nacional de Información de Gobierno, Seguridad Pública e Impartición de Justicia and Instituto Nacional de Estadística y Geografía (2022). This survey

provides, among other things, an official estimation of the dark figures of crime for the felonies from table 2 and many other illegal actions.

Felony	Cases per 100,000 inhabitants
Extortion	5050
Kidnapping	1982
Car theft	1775
Robbery	1801

Table 2. Some of the felonies experienced by the people in Michoacán in recent years. Source: Subsistema Nacional de Información de Gobierno, Seguridad Pública e Impartición de Justicia and Instituto Nacional de Estadística y Geografía (2022).

Hotelling's work has had many updates over the years. Among some recent iterations, Huang (2009), Lambertini (2002), and Matsushima (2001) considered spatial models to generalize the competition between the companies. The application we present in section 4.1 can be thought of as one. However, we drift away from the State-of-the-Art for both, our methods and our goals are different from the original ones. Indeed, although the firms look to maximize their market share, our main goal is to present an alternative against the avocado pirates. As for our methods, we state the problem in terms of the stochastic dynamic programming framework with a fixed number of competitors on an infinite horizon under the ergodic criteria, then reduce it using a mean-field result, and finally look for equilibrium in a set of probability measures.

Our contribution from section 4.1 can be in the gap between the works by de Melo, Frank, and Brantingham (2017); Lee and Lee (2007); Meyer (2010); Schunke, de Oliveira, and Villamil (2014); She et al. (2015), and the studies on the Hotelling competition model. The former references use Voronoi methods for law enforcement; while the latter describe how well-established players compete for market share with each other. We do not assume the presence of an authority responsible for watching over a proper rule of law but rather consider that the packaging facilities owners implement a solution on their behalf.

The problem at hand is so important, that researchers such as Boone, Skipper, Murfield, and Murfield (2016); Ekwall and Lantz (2018, 2020a, 2020b); King (2023); Micheal (2024) have described the modus operandi of freight hijackers and summarized some ways to avoid their pernicious effects. For instance, Liang, Fan, Lucy, and Yang (2022) examines the major risk factors that affect cargo theft from freight supply chains, creates a data-driven Bayesian network model to achieve risk diagnosis and prediction for cargo theft, assesses the important risk factors to forecast the likelihood of cargo theft incidents, examines actual incidents to validate the model, and offers recommendations for preventing cargo theft. Another relevant research paper is that by Cedillo-Campos, Flores Franco, and Covarrubias (2024). In addition to offering insights into how the physical Internet perspective can enhance supply chain resilience and provide information to the public and private decision-makers for creating more resilient and sustainable delivery systems, this paper puts forth a mathematical model to reduce the risk of cargo theft. Both our proposals resemble the latter two references in the assessment they make of the affections that crime produces in the supply chain of the avocado industry in Michoacán but turn out to be original because they are feasible for the structure of the industry in the state. All the previously mentioned methods would require a considerable (pecuniary, theoretical, or straightforward technological) effort to be effectively implemented in the state.

The approach we employ in section 4.1 differs from that of Huang (2009), for in this reference only pure strategies are used, while we consider strategies of mixed type. We also point out that in our model, the players are already in place when the game starts, and they play simultaneously, while Lambertini (2002) considers Stackelberg equilibria when competitors enter the system sequentially. Finally, our model resembles that of Matsushima (2001) in that they both study a two-stage problem. (In the latter, a circular city model of location and quantity choice is studied.) However, a critical difference is that here, the optimization criterion we study enables us to assert the existence of a Nash equilibrium in the long run for a Voronoi game on the plane, even though our set of actions is finite. In this sense, our first proposal resembles those of Ahn, Cheng, Cheong, Golin, and van Oostrum (2004) and Byrne, Fekete, Kalcsics, and Kleist (2023), for the case of three players. García-Meza and López-Barrientos (2016) present another relevant study for our developments. There, as in our study, there is a struggle for market share in an oligopoly, and there are entry barriers that inhibit the participation of more players. However, in the present paper, we work on the stochastic framework with discrete-time variables.

From a game-theoretical point of view, we argue that the results in section 4.1 ex- tend the work by López-Barrientos, Mendoza-Madrid, and González-Vega (2024) to a multiplayer context and represent the application of the theory presented by López- Barrientos, Escobedo-Trujillo, and Fernández-Arias (2024) on an infinite horizon. In- deed, while López-Barrientos, Mendoza-Madrid, and González-Vega (2024) study the definition and characterization of the ergodic criterion for Markov decision problems, López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2024) work with a nonzero- sum stochastic game defined on a set of probability measures. That is, the former reference deals with a basic criterion for degenerate games with another basic long-run criterion for two-person games, while the latter studies its extension to the realm of non-degenerate games. This fact enables us to state that the proposal from section 4.1 is specifically designed to be sustainable on an infinite horizon. Finally, we acknowledge the fact that Arapostathis, Biswas, and Carroll (2017) studied a problem where the governing dynamics of the interacting objects are controlled by a stochastic diffusion in many dimensions and provides the existence of a mean-field equilibrium under the ergodic criterion for ω players. In our case, the dynamic of interaction between the agents is observable but unknown to the central agents; so, we use a statistical estimation procedure to model the situation.

Black and Scholes (1973) developed a framework for finding fair prices of financial options under the assumption that the ratio of returns to underlying assets has a lognormal distribution. Later, Merton, Brennan, and Schwartz (1977) reformulated it and used it to find the price of the so-called contingent payments. On the other hand, Boyarchenko and Levendorskii (2002) showed how to value contingent payments under the usual hypotheses of a complete market, and they managed to carry out the corresponding sensitivity analyses without assuming that the distribution of the log-returns follows a Gaussian distribution. Fernández-Arias, López-Barrientos, and Moreno Ruíz-Esparza (2021) used the so called Ito isometry (Øksendal, 2003, Lemma 3.1.5 and Corollary 3.1.7), the methods proposed by Higham (2001) and Higham (2004), and the toll road seasonality analysis presented by Wright and Paquette (1987) to isolate two algorithms through which it is possible to define the premium to be paid in exchange for the right of property over the turnpike.

The proposal from section 4.2 intends to be a revision of the paper by Fernández- Arias et al. (2021), where a preliminary study of the subject is presented. The main assumptions there are that the logarithm of the underlying process is Gaussian, that there exist arbitrage opportunities and there are seasonal adjustments on the number of cars traversing the highway. While we intend to keep the latter hypothesis, we will use a Poisson process to relax the Gaussian property of the underlying process. As for the sustainability of this proposal in the long run, we argue that as long as the toll road manager has a way to effectively assess the numbers and classes of the users, the method will be feasible.

4. Methodology and results

4.1 Covering piracy acts hors-route

Our first proposal is oriented to the sixth phase of the logistics referred to in table 1: transportation to packaging facilities. Since most of the packaging houses in the municipality of Uruapan, many orchards send their product on long and risky journeys in the hope that it will eventually reach their destination (Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C., 2021). According to the interview we held with the businessman who asked for confidentiality, these journeys last up to four hours, and the owners of packaging facilities fail to see the advantages of placing new branches of their businesses. For the sake of simplicity, we will work with only three packaging firms rather than considering the existing 84 of them:

- Empaque Los Reyes S.A. de C.V. Located at: 19.699420716076524 N, 102.50184490239482 W.(Red marker in figure 3.)
- Avocados Esquivel S. de R.L. Located at: 19.526697782157427 N, 102.42147599085436 W. (Blue marker in figure 3.)
- Avoproduce México, S. de P.R. de R.L. de C.V. Located at: 19.4136483 N, 101.938 W. (Orange marker in figure 3.)

Remark 1. In figure 3, the orchards are uniformly distributed within the geographical coordinates provided by Gutiérrez-Contreras et al. (2010). Of course, we do this to illustrate our developments. An approximation to the actual (and current) distribution of the orchards can be found in the paper by Gutiérrez-Contreras et al. (2010).

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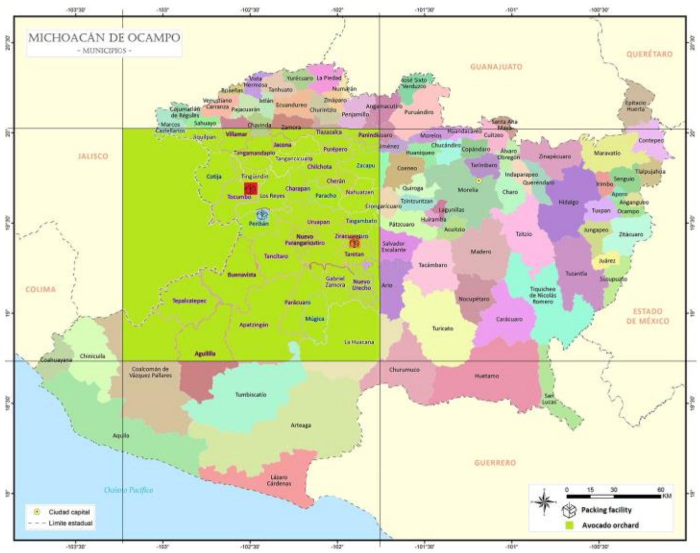

Figure 3. The municipalities where the avocado producers are in Michoacán. Taken from Juárez Navarro (2021) with information from Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C. (2021) and Gutiérrez-Contreras et al. (2010).

We will assume that each company has only a limited installed capacity to add new facilities to the map. For illustration, and to keep the calculations tractable, we will enable them to install up to one new packaging house. For $n = 1, \dots, 35000$, let $X_n^{35000}(t)$be a random variable that represents the status of the n-th orchard at time t = 0, 1, ... This random variable is defined on the probability space $(\Omega, \mathcal{F}, P)$, and takes on values from a finite set S, whose members are:

(0, v) – Shock due to accident or robbery while going to location v, (1)
(1, v) – Client of the first packaging company at location v, (2)
(2, v) – Client of the second packaging company at location v, (3)
(3, v) – Client of the third packaging company at location v, (4)

where $v \in [-103.216667, -101.783333] \times [18.75, 20.1]$, i.e., v is a point in the rectangle referred to in figure 3. To ensure that S is a finite set in our illustration, at each time, we allow v to take on only the coordinates of the available packaging facilities placed by the central players. With this in mind, the initial positions of the players are:

- $v_1(0) = (-102.50184490239482, 19.699420716076524)$
- $v_2(0) = (-102.42147599085436, 19.526697782157427)$
- $v_3(0) = (-101.938, 19.4136483)$
-

and there will be a moment when all of them will have one more. However, the pirates may catch the producers at any point in the route that leads from the orchards to these locations. With all of these in mind, the random variable can be in any of the $2 \times 3 \cdot (1 + 1) = 12$ states of the form (1)-

(4), where v is any of the (up to) six locations of the facilities installed by the central agents. Our central agents will (try to) control the process $(X_n^{35000}(t): t = 0,1, \dots)$ for n = 1, … ,35000. The agents will choose their actions $u_1(t), u_2(t), u_3(t)$ at each time t= 0, 1,... from given Borel sets U1, U2, U3, respectively. For us,

$$U_k = \{[-103.216667, -101.783333] \times [18.75, 20.1]\} \cup \{.\_.\}\backslash \text{ for k} = 1,2,3.$$

That is, the longitudes and latitudes, respectively, referred to in figure 3; or the special symbol "._." that stands for the action of not installing any new facilities (which might occur should the player decide not to add any new facilities, or should they have already placed their available facility). The idea behind this definition is that, for k = 1, 2, 3, when the k-th player chooses the action $u_k \in U_k \setminus \{.\_.\}$ then they build both: a packaging facility in uk, and the corresponding road from its closest facility to uk. For n = 1,2, … ,35000, the evolution of $(X_n^{35000}(t): t = 0,1, \dots)$ is given by:

$$X_n^{35000}(t+1) = \mathrm{F}\left(X_n^{35000}(t), u_1(t), u_2(t), u_3(t), \xi(t)\right), \text{ t} = 0,1,$$

where $\mathrm{F}: \mathrm{S} \times U_1 \times U_2 \times U_3 \times R \rightarrow S$ is a given (known) function and $(\xi(t): t = 0,1, \dots)$ is a sequence of independent and identically distributed Real random variables defined on $(\Omega, \mathcal{F}, P)$ with a common (but unknown) density ρ. In our case, we want the function F to mirror, at the same time, the preference of the n-th orchard for the closest packaging facility, and their uncertainty towards the possibility of being hijacked on the way there. To this end, let yn be the coordinates of the n-th orchard and define, for t = 0, 1, ..., the distance

$$\mathrm{D}\big(y_n, u_z(t)\big) := min\big\{d\big(y_n, u_1(\tau)\big), d\big(y_n, u_2(\tau)\big), d\big(y_n, u_3(\tau)\big): \tau = 0, \dots, t\big\}, \quad (5)$$

where $u_k(\tau)$ stands for the action taken by the k-th player at time $\tau = 0, \dots, t$; and, if $u_k(\tau) \in U_k \setminus \{.\_.\}$, then $\mathrm{d}\big(y_n, u_k(\tau)\big)$ is the Euclidean distance between y_n and $u_k(\tau)$; otherwise $\mathrm{d}\big(y_n, u_k(\tau)\big) = 0$.

Remark 2. (a) Observe that, since $u_1(0) = v_1(0), u_2(0) = v_2(0),\backslash\ \mathrm{u}_3(0) = v_3(0)$, then the distance referred to in (5) is finite and well-defined for all n =1, ..., 35000 and all t=0, 1, ...

(b) For n=1, ..., 35000, the symbol z in (5) stands for the index of the central player who has the closest packaging facility to the n-th orchard. We use these definitions and Remark 2 to finally let, for t = 0, 1, ..., and n =1,...,35000,

$$\mathrm{F}\big(X_n^{35000}(t), u_1(t), u_2(t), u(3), \xi(t)\big) = \big(z, u_z(t)\big)\backslash \text{ if}\backslash\xi(t) = 0, \text{ and}$$
$$\mathrm{F}\big(X_n^{35000}(t), u_1(t), u_2(t), u(3), \xi(t)\big) = (0, v)\backslash \text{ if}\backslash\xi(t) = 1,$$

where $\xi(t)$ is a Bernoulli random variable that acquires a unitary value if the orchard owner is hijacked on their way to the packaging facility, and a null value in another case. Obviously, we

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intend to have $\mathrm{P}(\xi(t)=1)=\mathrm{p}$ to be "large" if $\mathrm{D}\big(y_n,u_z(t)\big)$ overtakes a given barrier. Note that we do not assume any prior knowledge as to the value of this number whatsoever. All these facts enable us to assert that the distribution ρ of ξ(•) is unknown to us. When the central players select their actions $\big(u_1(t),u_2(t),u_3(t)\big)\in U_1\times U_2\times U_3$, a random movement of the objects from class i ∈ S to the class j ∈ S happens according to a joint transition probability:

$$K_{ij}^{\rho}(u_1,u_2,u_3)\coloneqq P(X_n^{35000}(t+1)=j\big|X_n^{35000}(t)=i,u_1(t)=u_1,u_2(t)=u_2,u_3(t)=u_3)$$

$$=\int_R I_j\big(F(i,u_1,u_2,u_3,\xi)\big)\rho(\xi)\mathrm{d}\xi,$$

which is homogeneous in n (see (2.1) in the paper by López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025) or (2.1) in that by López-Barrientos, Mendoza- Madrid, and González-Vega (2024)). To estimate the density of the random variable ξ, we use the information from table 2. It will give us that the probability that a citizen from Michoacán taken at random suffers at least an extortion, a kidnapping, a car theft, or other sorts of robbery during a year is of $\frac{5050+1982+1775+1801}{100{,}000}=$ 0.10608. With this in mind, we define the random variable N that counts the number of felonies suffered by a transporter during the year. If further, we assume that N follows Poisson's law, then we have that $\mathrm{P}(N\geq 1)$ = 0.10608, which means that P(N=0) = 0.89392. We use this result to obtain an intensity rate per annum of λ =0.112138993 for the felonies suffered by transporters in Michoacán. This number can be used to compute the probability that a transporter gets hijacked during a t-hour period with an Exponential approach: $1-\exp(-\lambda t)$. For example, we readily know that a transporter can be on the road for up to four hours. Then, the estimated probability that they suffer an assault is:

$$1-\exp\left(-0.112138993\cdot\frac{4}{24\times 365}\right)=0.0000512037. \qquad (6)$$

For k=1,2,3, let $c_k\colon \mathrm{P}(S)\times U_1\times U_2\times U_3\to[0,1]$ be the reward function that the k-th central agent intends to maximize. Here, P(S) stands for the set of probability measures defined on the state space S. This function represents the market share that the *k-th* central agent enjoys, and as it is only natural, it depends on the proportion of the orchards that chose the k-th player to pack their product, and on the actions taken by all three agents at each moment to try to approach them. Figure 4 shows a geometric construction representing the initial distribution of the market share between the central agents without considering the action of crime. To prepare for it, we applied the rule (5) to each point in the square region. Point A corresponds to the location marked in red in figure 3; point B, to that marked in blue in figure 3; and point C represents the packaging house marked in orange in figure 3. Note that △ABC is a graph whose dual is the (Voronoi) tessellation we are looking for (see Chapter 7 in the book by de Berg, Cheong, van Kreveld, and Overmars (2008)). To this end, we trace the bisectors to AB, AC, and BC and use the circumcenter to define the partition we are looking for.

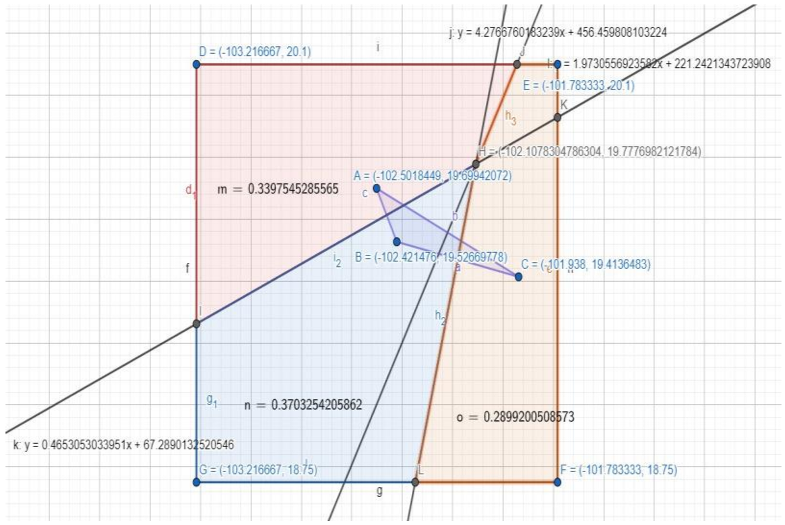

Figure 4. Initial distribution of the market shares without considering the pernicious action of crime.

Now we relate the model that yields (6) with the distance traveled by a transporter from the furthest orchard to its corresponding packaging facility. To this end, observe in figure 4 that the longest distance a transporter needs to travel to arrive at a packaging facility is from point B to point G: 1.11157 distance units. By a proportion argument, in general, the time spent on the road to go from point (u_1,v_1) to point(u_2,v_2) will be given by:

$$\frac{4}{1.11157}\sqrt{(u_1-u_2)^2+(v_1-v_2)^2}.$$

Interpreting the probability of being hijacked when going from an orchard to a packaging facility as the proportion of the goods that fall into the hands of the criminals, we get to know how much of the product ends up in possession of the pirates by computing:

$$\int_{18.75}^{19.2618}\int_{-103.217}^{\frac{y-456.4898}{4.2766}}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+102.42)^2+(y-19.526)^2}} 1-\mathrm{e}^{-0.1121\mathrm{t}}\mathrm{dtdxdy}\approx 0.145855,$$

$$\int_{19.2618}^{19.7777}\int_{\frac{y-67.2890}{0.4653}}^{\frac{y-456.4898}{4.2766}}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+102.42)^2+(y-19.526)^2}} 1-\mathrm{e}^{-0.1121\mathrm{td}}\mathrm{tdxdy}\approx 0.0162662.$$

The sum of these proportions yields that the hypothetical proportion of avocado in the blue region from figure 4 is 0.1621212. Analogously, we compute the proportion of the good that ends up in criminal hands in the red region from figure 4:

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$$\int_{19.2618}^{19.7769}\int_{-103.217}^{\frac{y-67.2890}{0.4653}}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+102.5018)^2+(y-19.6994)^2}} 1-\mathrm{e}^{-0.1121\mathrm{t}}\mathrm{dtdxdy} \approx 0.0406185,$$

$$\int_{19.7777}^{20.1}\int_{-103.2467}^{\frac{y-67.2890}{0.4653}}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+102.5018)^2+(y-19.6994)^2}} 1-\mathrm{e}^{-0.1121\mathrm{t}}\mathrm{dtdxdy} \approx 0.05224$$

This yields a proportion of 0.092865. Finally, we calculate the proportion of avocado that crime controls in the orange region from figure 4.

$$\int_{18.75}^{19.7777}\int_{\frac{y-456.4898}{4.2766}}^{-101.7833}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+101.938)^2+(y-19.4143)^2}} 1-\mathrm{e}^{-0.1121\mathrm{t}}\mathrm{dtdxdy} \approx 0.0461188\,;$$

$$\int_{19.7777}^{20.1}\int_{\frac{y-221.24213}{1.9730556}}^{-101.7833}\int_{0}^{\frac{4}{1.11157}\sqrt{(x+101.938)^2+(y-19.4143)^2}} 1-\mathrm{e}^{-0.1121\mathrm{td}}\mathrm{tdxdy} \approx 0.0144258.$$

That is 0.0605446. Table 3 displays the summary of the proportions in each state (1)-(4).

State	Proportion
(0, location A)	9.2865%
(0, location B)	16.2121%
(0, location C)	6.0545%
(1, location A)	24.6889%
(2, location B)	20.8205%
(3, location C)	22.9376%
Total	100%

Table 3. Initial distribution of the orchards among the packaging facilities and the criminals.

Now we define the discrete-time dynamic game associated with the 35000-orchard system previously introduced, through the following elements:

$$\Gamma_{35000} := \left(P(S), U_1, U_2, U_3, G_\rho^{35000}, c_1, c_2, c_3\right),$$

where G_ρ^{35000} defines the transition function of the process of market shares. Algorithm 2 in López-Barrientos, Escobedo-Trujillo and Fernández-Arias (2025) shows the details. This model describes the evolution of the system. At the time t=0,1..., the k-th central agent observes the vector of market share $\vec{m}(t) \in P(S)$, and then chooses an action $u_k = u_k(t) \in U_k$. As a consequence, this agent earns a reward $c_k(\vec{m}, u_1, u_2, u_3)$, and the system evolves to a new state $\overrightarrow{m'} = \overrightarrow{M^{35000}}(t+1) \in B$ according to the transition law:

$$Q_{\rho}(B|\vec{m}, u_1, u_2, u_3) := P\left(\overrightarrow{M^{35000}}(t+1) \in B \middle| \overrightarrow{M^{35000}}(t) = \vec{m}, u_1(t) = u_1, u_2(t) = u_2, u_3(t) = u_3\right) \quad (7)$$

with $B \in \mathcal{F}'$ a σ-algebra of events of $(P(S) \times U_1 \times U_2 \times U_3)^{\infty}$. Then, the process repeats itself and the one-stage rewards for the agents are accumulated through an expected reward criterion. The ultimate goal of the k-th central agent is to maximize the respective expected average reward given by:

$$J_k^{35000}(\pi^{35000}, \vec{m}) :=$$

$$\lim_{T\to\infty} \inf \frac{1}{T} E_{\vec{m}}^{\pi_1^{35000}, \pi_2^{35000}, \pi_3^{35000}} \left[\sum_{t=0}^{T-1} c_k\left(\overrightarrow{M^{35000}}(t), u_1(t), u_2(t), u_3(t)\right)\right], \quad (8)$$

where π_{ι}^{35000} stands for the sequence of actions taken along time by the ι-player, for $\iota = 1,2,3$; and $E_{\vec{m}}^{\pi_1^{35000}, \pi_2^{35000}, \pi_3^{35000}}[\cdot]$ is the conditional expectation of $[\cdot]$ given that the central players use the sequence of actions $(\pi_1^{35000}, \pi_2^{35000}, \pi_3^{35000})$, and the distribution of the market share is $\vec{m}$. Remark 3. The existence of the probability measure referred to in (7) is ensured by the theorem of Ionescu-Tulcea (see Chapter 5.4 in the book by Dynkin and Yushkevich (1979); or Proposition C.10 and Remark C.11 in the book by Hernández-Lerma and Lasserre (1996) and Proposition V.1.1 in the book by Neveu (1965). For more details, see the comments below Definition 3.1 in the paper by López-Barrientos, Mendoza-Madrid, and González-Vega (2024) and Remark 3.4 in the paper by López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025).

By applying standard dynamic programming arguments (see Proposition 3.10 in López-Barrientos, Mendoza-Madrid, and González-Vega (2024), and Proposition 3.17 in López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025)), it is possible to state the situation as trying to find a solution to the following system of Isaacs' inequalities By applying standard dynamic programming arguments (see Proposition 3.10 in López-Barrientos, Mendoza-Madrid, and González-Vega (2024), and Proposition 3.17 in López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025)), it is possible to state the situation as trying to find a solution to the following system of Isaacs' inequalities:

$$J_{*k}^{35000} + r_{*k}^{35000}(\vec{m}) \leq \max_{u_k \in U_k}\left[c_k(\vec{m}, u_1, u_2, u_3) + \int_{P(S)} r_{*k}^{35000}(y) Q_{\rho}(\mathrm{d}y|\vec{m}, u_1, u_2, u_3)\right], \quad (9)$$

for k = 1, 2, 3. Here, r_{*k} is a discrepancy function that measures the bias from the value at (Nash) equilibrium for the k-th player, and:

$$J_{*k}^{35000} := \sup_{\vec{m}, \pi_1^{35000}, \pi_2^{35000}, \pi_3^{35000}} J_k^{35000}(\pi_1^{35000}, \pi_2^{35000}, \pi_3^{35000}, \vec{m}).$$

Unfortunately, the integral in (9) is equivalent to

$$\underbrace{\int_0^1 \cdots \int_0^1 r_*^{35000}\left(G_\rho^{35000}(\vec{m}, u_1, u_2, u_3, w_1, \dots, w_{35000})\right)}_{35000 \text{ times}} \mathrm{d}w_1 \cdots \mathrm{d}w_{35000}$$

which turns this problem into a nearly unsolvable one. On the other hand, since the problem we have stated meets the conditions of the control problem from section 4.1 in López-Barrientos, Mendoza-Madrid, and González-Vega (2024), instead of trying to devise a way to solve (9), we set ω=3, use algorithm 1 (which is a modified version of Algorithm 2 in López-Barrientos, Mendoza-Madrid, and González-Vega (2024)) and solve the resulting mean-field game problem (see also López-Barrientos (2014)). Remark 4. The fact that the deterministic policies found through the deterministic mean-field game resulting from our approach is proved in sections 3 and 4 from the paper by López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025). The key to the proof is an Abelian theorem (which can be found in Theorem 1.1 in the paper by López-Barrientos, Escobedo-Trujillo, and Fernández-Arias (2025), and Lemma 5.6(a) in the book by Hernández-Lerma, Laura-Guarachi, Mendoza-Palacios, and González- Sánchez (2023)) and the so-called vanishing discount technique. See the references just cited for more details.

Algorithm 1. Policy iteration algorithm for the nonzero-sum mean-field game

Input: Estimated density $\hat{\rho}$ and tolerance level $0 < \varepsilon < 1$.

Output: For each $k = 1, \dots, \omega$, triplet $\left(j_*^k, r_*^k, (f_*^k)\right)$.

1. Select $(\pi_{-1}^1, \dots, \pi_{-1}^\omega)$ and set $j_{-1}^k = -\infty$ for $k = 1, \dots, \omega$
2. $\iota \leftarrow 0$
3. Do
4. For k← 1 to ω
5. Find a constant j_ι^k such that $\left| j_\iota^k - c_k\left(\overrightarrow{m^{\pi_{\iota-1}^k, \dots, \pi_{\iota-1}^\omega}}(t), \pi_{\iota-1}^1, \dots, \pi_{\iota-1}^\omega\right) \right| < \varepsilon^t$ for all $t \geq 0$
6. Find an upper semi-continuous function $r_\iota^k \colon P(S) \to R$ such that
   $$j_\iota^k + r_\iota^k(\vec{m}) \leq c_k(\vec{m}, \pi_{\iota-1}^1, \dots, \pi_{\iota-1}^\omega) + r_\iota^k\left(G_{\hat{\rho}}(\vec{m}, \pi_{\iota-1}^1, \dots, \pi_{\iota-1}^\omega)\right) \text{ for all } \vec{m} \in P(S)$$
7. Find a policy π_ι^k such that
   $$c_k(\vec{m}, \pi_{\iota-1}^1, \dots, \pi_\iota^k, \dots, \pi_{\iota-1}^\omega) + r_\iota^k\left(G_{\hat{\rho}}(\vec{m}, \pi_{\iota-1}^1, \dots, \pi_\iota^k, \dots, \pi_{\iota-1}^\omega)\right)$$
   $$= \sup_\psi \left[c_k(\vec{m}, \pi_{\iota-1}^1, \dots, \psi, \dots, \pi_{\iota-1}^\omega) + r_\iota^k\left(G_{\hat{\rho}}(\vec{m}, \pi_{\iota-1}^1, \dots, \psi, \dots, \pi_{\iota-1}^\omega)\right)\right]$$
8. $\iota \leftarrow \iota + 1$
9. Next k
10. While $\left|j_{\iota-1}^k - j_\iota^k\right| \geq \varepsilon$ for some $k = 1, \dots, \omega$
11. Return $(j_\iota^k, r_\iota^k, \pi_\iota^k)$ for all $k = 1, \dots, \omega$

Now we execute algorithm 1. The geometric and graph-theoretic procedure we used to obtain table 3 yields the transitions between states at equilibrium in our problem. Indeed, the work of Byrne et al. (2023) and the references therein prove that the action that maximizes the supremum from line 7 in algorithm 1 (and therefore (8)), i.e. the market share, of the k-th agent at each time is to find the farthest orchard in the region under the control of the k-th agent without considering the pernicious action of crime (i.e. as in figure 4) and place a packaging house midway to that location.

The resulting tessellation after the action of the central agent at location B is shown in figure 5. Note that their action was to place a packaging house at location M. Figure 5 displays the zones controlled by each of the players: player A controls the region within vertices NSDJH; player B, the one within SNHRW G; and player C, EFW RHJ. An analogous procedure to the one that gave us table 3 gives table 4.

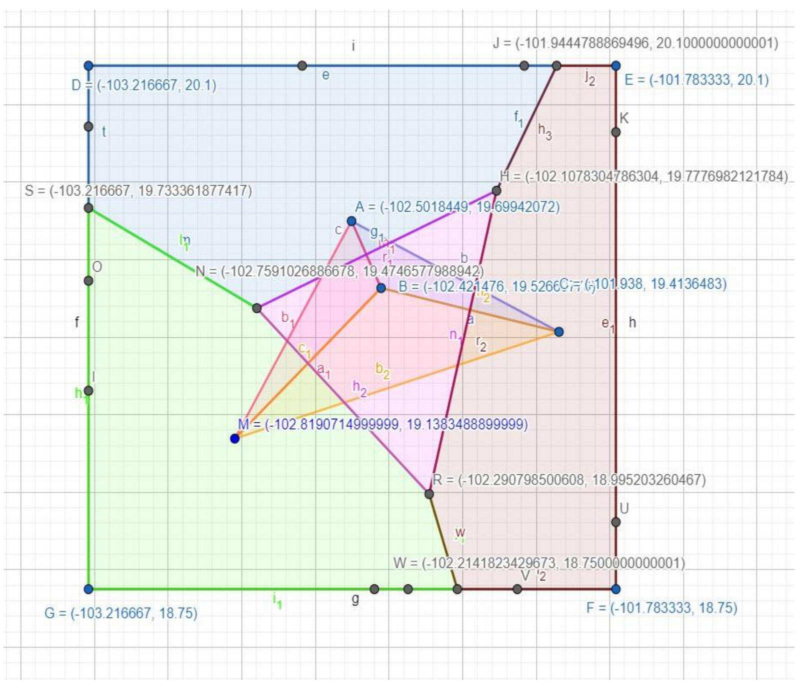

Figure 5. Distribution of the market shares after the action of player B without considering the pernicious action of crime.

According to line 5 in algorithm 1, the action taken by the player located at B yields the following average value for them:

$$j_1^B = 10.8031\% + 27.0924\% = 37.8955\%. \quad (10)$$

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Observe tables 3 and 4. At least one of the players located at A and C cannot exercise action ".__." if B plays M. Then, the player located at A should exercise their optimal response by placing a packaging facility at point B_1. The result can be seen in figure 6.

State	Proportion
(0, location A)	6.5345%
(0, location B)	0.9326%
(0, location M)	5.3237%
(0, location C)	5.5372%
(1, location A)	22.5021%
(2, location B)	10.8031%
(2, location M)	27.0924%
(3, location C)	21.2744%
Total	100%

Table 4. Distribution of the orchards among the packaging facilities and the criminals after the action of the packaging facility located at B.

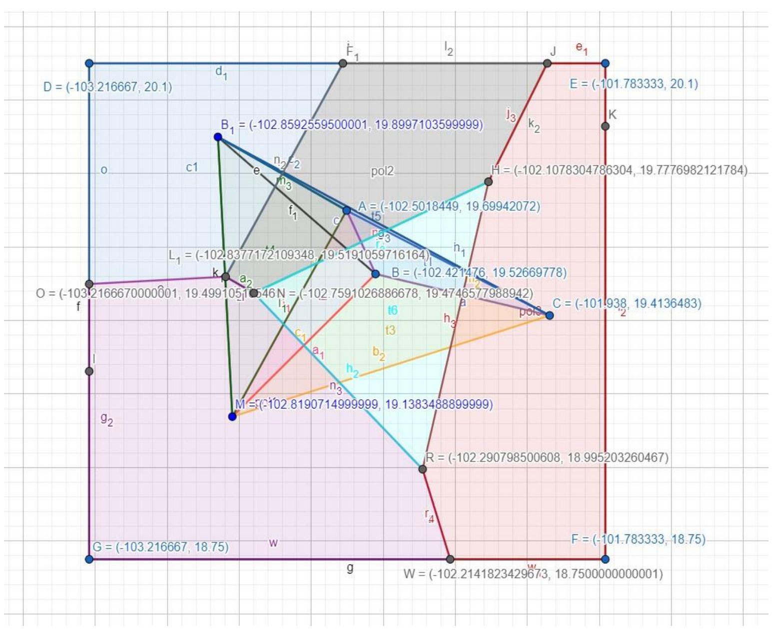

Figure 6. Distribution of the market shares after the action of players A and B without considering the pernicious action of crime.

Table 5 shows the effect of the player's A move. It also makes it clear that player C should not play ".__.".

State	Proportion
(0, location A)	1.9219%
(0, location B1)	1.5069%
(0, location B)	0.9326%
(0, location M)	4.1716%
(0, location C)	5.5372%
(1, location A)	12.9503%
(1, location B1)	14.9514%
(2, location B)	10.8031%
(2, location M)	25.9507%
(3, location C)	21.2744%
Total	100%

Table 5. Distribution of the orchards among the packaging facilities and the criminals after the action of the packaging facility located at A.

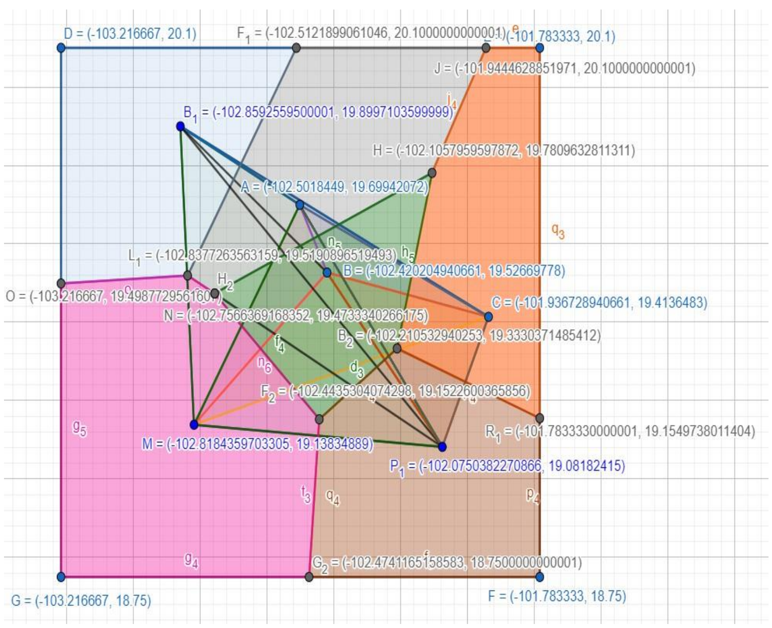

Figure 7. Distribution of the market shares after the action of players A, B and C without considering the pernicious action of crime.

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According to line 5 in algorithm 1, the action taken by the player located at A yields the following average value for player B:

$$j_2^B = 10.8031\% + 25.9507\% = 36.7538\%. \quad (11)$$

Comparing this result with (10), one can tell that the action taken by the player located at A affected the market share of its competitor. While all of this is happening, the player located at C exercises their optimal play by placing a packaging house at the point labeled as P_1 on figure 7. The corresponding distribution of the market shares among established players and criminals is displayed in table 6.

State	Proportion
(0, location A)	1.9219%
(0, location B1)	1.5069%
(0, location B)	0.7322%
(0, location M)	3.1218%
(0, location C)	2.0768%
(0, location P1)	1.5264%
(1, location A)	12.9564%
(2, location B1)	14.9514%
(2 location B)	9.3640%
(2, location M)	23.6567%
(3, location C)	12.55237%
(3 location P1)	15.6300%
Total	100%

Table 6. Distribution of the orchards among the packaging facilities and the criminals after the action of the packaging facility located at C.

With the information from tables 3 and 6, we can compare the market shares before and after the players played their available optimal actions in the game we propose under the long-run ergodic criterion. The results are displayed in table 7. In particular, the second row of this table, (10) and (11) give the evolution of for ι = 1, 2, 3.

After the players have exhausted the possibility of placing new packaging houses, they should continue to play the action "._." ad infinitum. However, the result of the stochastic game under the ergodic payoff criterion (8) will not vary. This is a powerful argument given the sustainability of the method in the long-run. The owners of the 84 packaging facilities in Michoacán refuse to place any more branches because they do not see the benefit of doing so. Should a proper cost-benefit analysis of our proposal be available, we believe our results would encourage such an investment. Now we sketch the basics of such an assessment. According to the Asociación de Productores y Empacadores Exportadores de Aguacate de México A.C. (2023); Seriñá Garza (2022), the phases of avocado packaging are as in table 8. The estimated cost of the tasks referred to in table 8 depends on the commitment of the entrepreneurs to concepts such as circular economics and biobased packaging.

Player	Initial market share	Market share at equilibrium
Empaque Los Reyes	24.6888%	27.9112%
Avocados Esquivel	20.8205%	33.0206%
Avopoduce México	22.9376%	28.1823%

Table 7. Comparison of the respective market shares at the beginning of the game, and after the players have exhausted their optimal actions.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Avocado arrival from orchards	■														
2	Unloading of boxes of avocados		■													
3	Automatic emptying of boxes			■												
4	Removal of leaves and twigs				■											
5	Brushing process					■										
6	Fruit alignment						■									
7	Electronic photo taking							■								
8	Fruit labeling								■							
9	Selection of different qualities									■						
10	Packaging modules										■					
11	Box assembling									■						
12	Box distribution line										■					
13	Avocado packaging											■				
14	Surveillance process	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
15	Prerrefrigeration chamber												■			
16	Refrigeration chamber													■		
17	Plague control in the truck box													■		
18	Loading onto the truck box														■	
19	Placing safety bands on the load															■
20	Preparation of avocado sample for phytosanitary inspection															■

Table 8. Gantt chart depicting the avocado packaging process according to Seriñá Garza (2022).

For instance, according to Matthews, Moran, and Jaiswal (2021), by 2030 in the European Union, all the plastic materials used in the packaging process should be either reusable or susceptible to being recycled cost-effectively; but paradoxically, biobased and biodegradable plastics are currently more expensive than fossil-based plastics on a weight basis. Nevertheless, Halonen et al. (2020); Michaliszyn-Gabry´s et al. (2022), among others, study some specific material properties that can allow for cost reductions in the use phase. The bottom line here is the fact that a proper cost-benefit analysis of the proposal is beyond the scope of this paper, for it requires an adequate assessment of the willingness of the community to engage in green activities in their hypothetical new facilities. However, table 8 sheds some light on the matter and represents a suitable point of departure for such a study.

4.2 Funding insurance against theft of goods on the highway

In 2023, the governmental authorities from Michoacán authorized the concessionaire enterprise to increase the number of lanes on the 66.8 km-long toll highway from Pátzcuaro to Uruapan. Indeed, the civil organization Movimiento estatal de reacción ciudadana y estabilidad social (see tumereces.org) convinced the authorities to ask the concessionaire to modernize the highway to provide the population with a safe road that connects some of the most important cities in the state. According to Coordinación General de Comunicación Social (2023) and Gobierno de México (2023), the project has an estimated cost of over 161 million USD. Table 9 shows the average daily traffic (ADT) data from each entry point in the toll road.

Unfortunately, even on these modernized roads, criminals have found ways to affect transporters. The ADT can help us price hedging instruments against these phenomena in the fashion of the paper by Fernández-Arias et al. (2021). We assume that the highway manager would be interested in offering insurance to their users in general, but to avocado transporters in case they suffer theft of goods in transit on the highway. To transfer the risk, they would buy coverage from a third party in exchange for a (fair) premium we propose to cover its risk using a correlated variable the ADT. Hence, this proposal is directed to the eighth phase referred to in table 1: transportation to export ports or domestic transportation.

We suppose that when the ADT goes below a certain number (barrier B) it is more likely to suffer a robbery. This implicitly suggests a high and negative correlation between ADT and the rate of robberies. We propose to compute the fair price of such payment by presenting the pseudo-code and calibration to value a financial instrument that replicates the phenomenon under the assumption that the Daily Traffic flow on the highway is a Poisson random variable (see page 107 the book by Wright and Paquette (1987). To this end, we propose algorithm 2, which is a modification of Algorithm 2 in the paper by Fernández-Arias et al. (2021). Generating Poisson random variables is a very well-known and standard procedure in probability theory. To this end, we use algorithm 2 from the paper by López-Barrientos, Zayat-Niño, Hernández-Prado, and Estudillo-Bravo (2022).

To execute algorithm 2, we let r be the current Mexican risk-free rate of 11.48% and set the barrier B to 20360 x 20% = 4072. That is, to the highest ADT of trucks from table 9. (We defend this hypothesis by arguing that the numbers from table 9 precede the rehabilitation from the year 2023.) We also assert that a deliberate prize of E =106 Mexican peso is immediately paid if the daily stock of trucks on the highway ever falls below the level of B (we consider a price per ton of avocados of about 70000 Mexican peso). We also let the traffic intensity of trucks λ be the center of mass of the ADT of trucks along the turnpike, and thus we take λ =2546.82. To see how we got this number, it suffices to use table 9 to take the ADT of trucks in each section of the toll road and take the weighted average of trucks per kilometer. We also assume the seasonal coefficients from table 10.

From	To	Distance	ADT	Trucks
Pátzcuaro (19.5137, - 101.5734)	Huerta Mi Bonita	13.5km	7418	20.9%
Huerta Mi Bonita (19.4930, - 101.6097)	Plaza de cobro Zirahuén	5.5km	6688	18.8%
Plaza de cobro Zirahuén (19.5056, - 101.6594)	Santa Clara del Cobre	12km	11699	24.7%
Santa Clara del Cobre (19.4631, - 101.7561)	Juju´cato	7.4km	10322	19.9%
Juju´cato (19.4341, - 101.8157)	Ziracuaretiro 1	8.5km	10430	22.9%
Ziracuaretiro 1 (19.4360, - 101.8895)	Ziracuaretiro 2	4.7km	10220	23.3%
Ziracuaretiro 2 (19.4205, - 101.9272)	Lázaro Cárdenas	3.1km	18683	20.1%
Lázaro Cárdenas (19.4165, - 101.9548)	Uruapan (19.4064, - 102.0422)	12.1km	20360	20%

Table 9. Sections of the highway from Patzcuaro to Uruapan. Source: Dirección General de Carreteras (2023).

October	0.89415	November	0.96435
December	1.09077	January	0.94472
February	0.86806	March	0.95401
April	1.14642	May	1.00693
Jun	0.97059	July	1.16154
August	1.05416	September	0.94429

Table 10. The seasonal coefficients for the next year. **Source**: Fernández-Arias et al. (2021).

We simulate 100,000 times on a one-year horizon and obtain a fair price of 117,693.70 Mexican peso for the instrument, which lies in the 95%-confidence interval 117691.4, 117969.01 . Note that algorithm 2 gives the probability of exercise of this earnout. Indeed, taking r =0 and E =1 in algorithm 2 yields a probability of exercise of 13.39% (cf. the paper by Fernández-Arias et al. (2021) for some more details). This should encourage investors to partake in the enterprise of funding such insurance.

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Algorithm 2. Computation of a 95% confidence interval for the fair price of the down-and-out option with single arrival of American type.

Input: Number of simulations to perform M; time horizon T; barrier to cross/reach at least one time B; number of evaluations of the barrier L; monthly daily average volume at the initial time S0; strike price E; discount rate r; Poisson intensity λ; vector of seasonal coefficients R. Output: 95%-confidence interval for the instrument's value at time t=0.

1. Compute the step size $\Delta t \leftarrow \frac{T}{L}$
2. For $i \leftarrow 1$ to M
3. $I \leftarrow 0$
4. $j \leftarrow 1$
5. For l<1 to $j \leq 1$
6. Compute a realization of the random variable $\xi_j \sim \text{Poisson}(\lambda \cdot j \cdot \Delta t)$
7. $S_{j+1} \leftarrow \xi_j$
8. $\overline{S_{j+1}} \leftarrow L \cdot \frac{R_j}{\sum_{k=1}^{L} R_k} \cdot S_{j+1}$
9. If $\overline{S_{j+1}} \leq B$ then
10. $I \leftarrow I + 1$
11. End if
12. Next l
13. $j \leftarrow j + 1$
14. If $I \geq 1$ then
15. $V_i \leftarrow e^{-rT} E$
16. Else
17. $V_i \leftarrow 0$
18. End if
19. Next i
20. $a_M \leftarrow \frac{1}{M} \sum_{i=1}^{M} V_i$
21. $b_M^2 \leftarrow \frac{1}{M-1} \sum_{i=1}^{M} (V_i - a_M)^2$
22. Return$\left[a_M - 1.96 \frac{b_M}{\sqrt{M}}, a_M + 1.96 \frac{b_M}{\sqrt{M}}\right]$

5. Conclusions and recommendations

This paper presents two proposals oriented to mitigate the risk of suffering losses because of crime in the Mexican state of Michoacán. The first is a method to maximize the market share of the owners of packaging houses in the state on a Voronoi game. We assume that these facilities are located off the existing toll roads. As a by-product of our results, we have shown a way to estimate the probability that the transporters suffer criminal action on their way toward the packaging facilities. From a societal point of view, this initiative turns out to be very attractive, for it represents a peaceful alternative to the violent actions taken by some locals in the presence of organized crime. As a by-product of both proposals presented here, their implementation would boost the level of legal employment in the entity, for none of them is currently being considered as a potential solution to the hijacking problem experienced by those in the avocado industry.

The second proposal is a method to finance an insurance schema using an instrument priced with the aid of the theory of contingent claims. It is oriented to the concessionaires in charge of the management of the highways in the state, and similar companies.

The example presented in section 4.1 represents a very partial answer to the piracy and smuggling problem that the producers from the Mexican state of Michoacán face daily. Indeed, our research is restricted by the limitations of the two-dimensional view of the geography displayed in figure 3, as it does not consider any geographic accidents, such as rivers, elevations, or depressions. See figure 8 and compare it to figure 7. The difference is notorious.

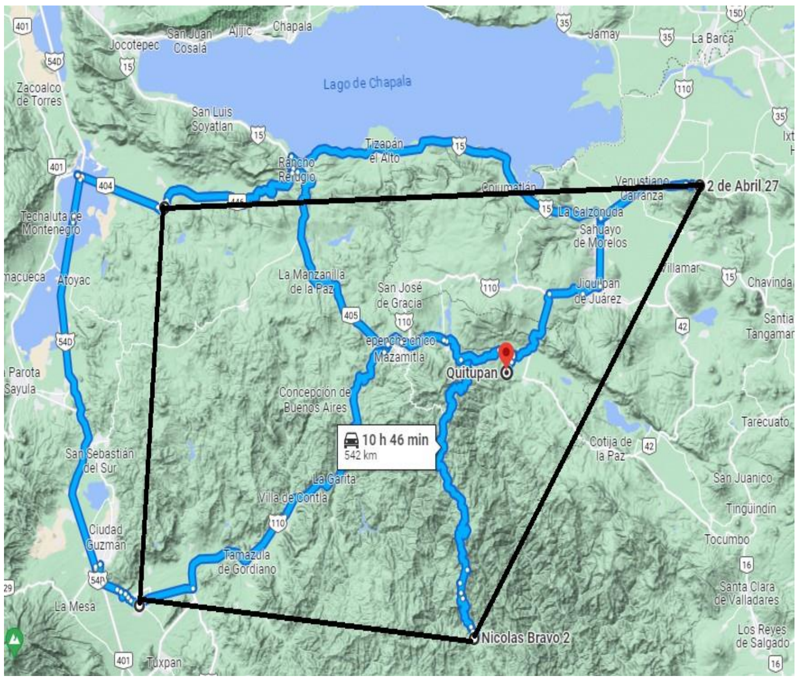

Figure 8. Actual location of the cities that conform the region DOL_1F_1 in figure 7. Prepared with information from The Google Maps Team (2023).

Moreover, for the sake of illustration, we have assumed that the distribution of the orchards in the land is uniform, but this hypothesis is not accurate (see remark 1). In the same thread of thought, we have willingly reduced the number of packaging companies to only three. Although table 8 shows the process of emballage, another limitation of our research is the fact that it only sketches a cost-benefit analysis of the proposal. A thorough assessment requires knowledge of the level of commitment to green actions on behalf of the local decision-makers. However, this information is not available at the time. For the same reason, the focus of the current study fails to consider the environmental impact of its proposals. We are readily working on a more complete version of the solution presented in this paper.

From a theoretical point of view, an advantage of the example we presented in this paper is that it illustrates a realistic application of our methods, where the actions available for the players belong to a non-denumerable infinite set and there are more than just two central agents. As for the method itself, we must emphasize the fact that this study presents both: a proper definition of the ergodic criterion in a stochastic context, and a way to obtain saddle points for this problem.

As for the applications of the problem to the State-of-the-Art, we are aware of the unrealistic assumptions on the distribution of the orchards in the state of Michoacán, and on the number of owners of packaging facilities in the state. We are readily working on these problems. The solution we are developing considers the orographic situation and the hydrography of the state, as well as all the packaging companies of the zone. Finally, this study recommends the concessionaire of highways in the state issue insurance against theft of goods and finance it by means of the procedure alluded to in section 4.2. Another one of the limitations of our results in this area is that we deliberately chose the values with which we executed algorithm 2.

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